The cancellation of worldsheet anomalies in the D=10 Green–Schwarz heterotic string sigma–model[*]

K. Lechner and M. Tonin

Dipartimento di Fisica, Università di Padova

and

Istituto Nazionale di Fisica Nucleare, Sezione di Padova

Italy

Abstract

We determine the two–dimensional Weyl, Lorentz and κ–anomalies in the $D = 10$ Green–Schwarz heterotic string sigma–model, in an $SO(1,9)$-Lorentz covariant background gauge, and prove their cancellation.

1. Introduction

The principal advantage of the Green–Schwarz string, with respect to the Neveu–Schwarz–Ramond formulation, is its manifest target space–time supersymmetry; its principal disadvantage is caused by the fundamental κ–symmetry which, being infinitely reducible, gives rise to problems with the Lorentz–covariant gauge–fixing of this symmetry. This last feature makes it rather difficult to compute the full worldsheet anomalies of the Green–Schwarz string, and the related sigma–model, and to prove finally their cancellation in $D = 10$. To do that, most papers, refs. [1,2], used a non covariant, semi–light cone gauge. However, a direct calculation of the contribution of the fermionic string fields ϑ to the Weyl anomaly in this gauge leads to a result which is $1/4$ of the correct value [1]. Possible ways to overcome this difficulty have been proposed in [2].

To our knowledge there is actually only one paper, ref.[3], by P.B. Wiegmann, in which the conformal (Weyl) anomaly in the Green–Schwarz heterotic string has been determined in a "covariant semi–light cone gauge", which is accessible in an on–shell configuration of the string, and shown to vanish in ten dimensions.*

The present paper can be viewed as an extension of the technique used in [3] in the following directions: first of all we determine the worldsheet anomalies in the heterotic string Green–Schwarz *sigma–model*, using the background field method combined with a normal coordinate expansion [5, 6]. This permits us to keep $SO(1,9)$ covariance manifest. The splitting of the string variables in classical and quantum fields, the classical fields being on shell, makes the covariant semi–light cone gauge accessible. Moreover, we determine the complete (Weyl, Lorentz and κ)–worldsheet anomaly due to the string coordinates (X, ϑ) and to the ghost (b, c)–system, and show that it cancels against the anomaly due to the 32 heterotic fermions. Finally, our procedure establishes a deep connection existing between the worldsheet anomaly and the target space $SO(1,9)$ Lorentz–anomaly, whose understanding is, actually, crucial for the cancellation of the total worldsheet anomaly.

Another important feature of our procedure is that it can be extended to

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* The correct result is also obtained in the framework of the problematic Lorentz covariant gauge fixing involving an infinite tower of ghosts [4].

other p–brane σ–models. In particular in [7] this method is applied to compute the worldvolume anomalies of the super fivebrane sigma–model, which is supposed to be dual to the $D = 10$ heterotic string, [8].

2. The action and the gauge fixing

The sigma–model action for the heterotic Green–Schwarz string in ten target space–time dimensions is given by

$$I = -\frac{1}{2\pi\alpha'} \int d^2\sigma \left(\frac{1}{2}\sqrt{g}\, g^{ij}V_i^a V_{ja} + \frac{1}{2}\varepsilon^{ij}V_i^A V_j^B B_{BA} - \frac{1}{2}\sqrt{g}\, e_-^j \psi(\partial_j - A_j)\psi \right).$$

(1)

The string fields are the supercoordinates $Z^M = (X^m(\sigma), \vartheta^\mu(\sigma))$, the 32 heterotic fermions $\psi(\sigma)$ and the worldsheet zweibeins $e_\pm^i(\sigma), g^{ij} = e_+^{(i}e_-^{j)}$. The induced zehnbeins are given by $V_i^A = \partial_i Z^M E_M{}^A(Z)$ and the $SO(1,9)$ flat index $A = (a, \alpha)$ stands for ten bosonic ($a = 0, ..., 9$) and sixteen fermionic ($\alpha = 1, ..., 16$) entries. The induced target space connections are $\Omega_{ia}{}^b(Z) = V_i^C \Omega_{Ca}{}^b$ for $SO(1,9)$, and $A_i(Z) = V_i^C A_C$ for the gauge group $SO(32)$. Flat two–dimensional light–cone indices for a worldsheet vector W_i are introduced via $W_\pm = e_\pm^i W_i, V_\pm^A = e_\pm^i V_i^A, \partial_\pm = e_\pm^i \partial_i$ etc. (see ref. [6] for the notation). Two–dimensional flat *vector* indices are indicated by an index with a "hat", $\widehat{a}$, e.g. $e_{\widehat{a}}^i$ ($\widehat{a} = 0, 1$). The action (1) is invariant under the transformations

$$\delta Z^M = \Delta^\alpha E_\alpha{}^M + c^i \partial_i Z^M$$

$$\delta\psi = \left(\frac{1}{2}(\ell + \lambda) + \Delta^\alpha A_\alpha + c^i \partial_i \right)\psi$$

$$\delta e_+^i = (\lambda + \ell)e_+^i - 4e_-^i \left(V_+^\alpha - \frac{1}{2}\psi\chi^\alpha\psi \right)\kappa_\alpha + c^j \partial_j e_+^i - \partial_+ c^i$$

$$\delta e_-^i = (\lambda - \ell)e_-^i + c^j \partial_j e_-^i - \partial_- c^i.$$

(2)

We indicate the ghost fields for worldsheet Weyl, Lorentz, diffeomorphism and κ–transformations respectively with λ, ℓ, c^i and $\kappa_\alpha \equiv \kappa_{+\alpha}$; $\Delta^\alpha = (\Gamma^a)^{\alpha\beta}V_{-a}\kappa_\beta$. $\chi^\alpha(Z)$ is the ten dimensional gluino superfield. Invariance under κ–transformations is, actually, achieved if the target space fields satisfy suitable superspace constraints, given in the Appendix. There we report also the BRST transformations of the ghost fields which, together with (2), give rise to a BRST operator Ω which closes if the string fields satisfy their equations of motion: $\Omega^2 = 0$ (on–shell).

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Having such an operator is extremely useful in that it allows to determine the κ–anomalies, once the Weyl and $SO(1,1)$–anomalies are known, upon enforcing the Wess–Zumino consistency condition (see below).

To compute the total anomaly we proceed as follows. We use the background field method along with a normal coordinate expansion [5,6], writing $Z = Z_0 + \Pi(Z_0, y)$ and $\psi = \psi_0 + \psi_q$, where (Z_0^M, ψ_0) are external "classical" fields, and ψ_q and the normal coordinates $y^A = (y^a, y^\alpha)$ are "quantum" fields over which we perform the functional integration. The functions Π^M trigger the manifest $SO(1,9)$ Lorentz covariance of the background field method. Since we choose to maintain the effective action diffeomorphism invariant, at the expense of local worldsheet Weyl–Lorentz anomalies, the ghost fields c^i (and the antighosts b_{ij}) can be treated as purely "quantum"; the zweibeins $e_\pm^i$ as well as the ghosts ℓ, λ, κ are considered as purely "classical". The classical fields transform according (the classical counterparts of) eq. (2). Moreover from now on, the pullback zehnbeins V_i^A, the Lorentz connection Ω_{iab}, the gauge connection A_i etc. will be the classical ones, i.e. will be evaluated at $Z = Z_0$. Finally, we set the classical fields on shell.

Since the action is invariant under (2) and $\Omega^2 = 0$ on–shell, even if the heterotic fermions are absent, we can derive first the anomaly $\mathcal{A}_1$, gotten by the functional integration over (y^A, b, c) for $\psi_0 = 0$. The dependence on ψ_0 of this anomaly can be retrieved by enforcing the Wess–Zumino consistency condition on $\mathcal{A}_1, \Omega\mathcal{A}_1 = 0$. Then we perform the functional integration over ψ_q, compute the related anomaly $\mathcal{A}_2$ and show that $\mathcal{A}_1 + \mathcal{A}_2 = 0$.

The core of the present paper is constituted by the $SO(1,9)$–covariant functional integration over the fermionic y^α which we perform below.

For $\psi_0 = 0$ the equations of motion of the metric in terms of classical fields becomes

$$V_i^a V_{ja} = \frac{1}{2} g_{ij} V_+^a V_{a-} \equiv e^{2\phi} g_{ij}. \tag{3}$$

The use of eq. (3) allows to perform a Lorentz–covariant κ–gauge fixing on the y^α. We define the matrix

$$\Gamma^\alpha{}_\beta = \frac{1}{2} e^{-2\phi} \frac{\varepsilon^{ij}}{\sqrt{g}} V_i^a V_j^b (\Gamma_{ab})^\alpha{}_\beta$$

which, due to (3), satisfies $\Gamma^2 = \mathbb{1}$, $tr\Gamma = 0$. Since the κ–transformation law for

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y^α is $\delta_\kappa y = \mathcal{V}_-\kappa_q + o(y)$, where κ_q is the quantum ghost, the condition

$$\frac{\mathbb{1}+\Gamma}{2}y = \frac{e^{-2\phi}}{4}\mathcal{V}_-\mathcal{V}_+y = 0 \tag{4}$$

eliminates just 8 of the 16 $y's$ and fixes κ–symmetry. Moreover, in the gauge (4), being algebraic, the ghost–fields κ_q do not propagate.

A second essential ingredient we need is the knowledge of the target space $SO(1,9)$ Lorentz anomaly of the effective action, which is due to the non invariance of the integration measure $\int\{\mathcal{D}y\}$ under local $SO(1,9)$ transformations.

In a non–covariant gauge this anomaly has been computed in ref. [6]; the techniques used there can be adapted for the gauge (4) and the result is *

$$\mathcal{A}(L) = \frac{1}{8\pi}\int d^2\sigma\sqrt{g}\ tr(\partial_- L\widetilde{\Omega}_+). \tag{5}$$

$L \equiv L_{ab}$ is the infinitesimal $SO(1,9)$ transformation parameter and $\widetilde{\Omega}_+{}^{ab}$ is defined by

$$\widetilde{\Omega}_+{}^{ab} = \Omega_+{}^{ab} - e^{-2\phi}V_+^{[a}D_+V_-^{b]} - T_+{}^{ab}, \tag{6}$$

$$D_\pm = \partial_\pm \pm \omega_\pm + \Omega_{\pm *}{}^*, \tag{7}$$

where $\omega_\pm$ are the worldsheet connections

$$\omega_\pm = \pm\frac{1}{\sqrt{g}}\ \partial_j(\sqrt{g}\ e_\pm^j), \tag{8}$$

in terms of which the scalar curvature becomes $\mathcal{R}^{(0)} = D_-\omega_+ - D_+\omega_-$.

The explicit expression of $T_+{}^{ab}$ is given in the Appendix, here it suffices to know that

$$T_{+ab}V_-^b = 0 \ . \tag{9}$$

With the symbol tr we indicate the trace in the vector representation of $SO(1,9)$ or of $SO(1,1)$ since no confusion should arise.

Under *finite $SO(1,9)$* transformations, with transformation parameter $\Lambda^a{}_b$, the measure $\{\mathcal{D}y\}$ changes, according to (5), by a Wess–Zumino action given by

$$\Gamma_{WZ}(\Lambda) = \frac{1}{8\pi}\left(\int d^2\sigma\sqrt{g}\ tr\left(\Lambda^T\partial_-\Lambda\widetilde{\Omega}_+ - \frac{1}{2}g^{ij}\partial_i\Lambda^T\partial_j\Lambda\right) - \frac{1}{3}\int_{D_3} tr\left(d\Lambda\Lambda^T\right)^3\right), \tag{10}$$

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* The non trivial part of the anomaly is clearly independent on the gauge–fixing.

where $\Lambda^a{}_b$ satisfies $\Lambda^a{}_b \Lambda^c{}_d \eta^{bd} = \eta^{ac}$. D_3 is a three–dimensional manifold with the string worldsheet as boundary.

3. The anomaly computation

Now we are able to perform the functional integration over the y^α. After normal coordinate expansion the relevant contribution to the expanded action is given by the $SO(1,9)$–invariant kinetic term for the y^α

$$\frac{1}{4} \int d^2\sigma \left(\sqrt{g}\, g^{ij} + \varepsilon^{ij} \right) \, y \, \Gamma_a V_i^a D_j y$$

which, upon enforcing (4), can be written as

$$I(V, \Omega, y) = \frac{1}{2} \int d^2\sigma \sqrt{g}\, g^{ij} V_i^a \, y \, \Gamma_a \frac{\mathbb{1} - \Gamma}{2} D_j \frac{\mathbb{1} - \Gamma}{2} \, y \tag{11}$$

where $D_j \equiv \partial_j - \frac{1}{4} \Gamma_{cd} \Omega_j{}^{cd}$.

The normal–coordinate–expanded action contains actually additional terms quadratic in the y^α; for what concerns the $SO(1,9)$ anomaly (5) these terms give just rise to the contribution proportional to T_{+ab} in eq. (5), while they do not contribute to the worldsheet Lorentz and Weyl anomalies.

The principal problem related with (11) is that its kinetic term is not canonical in the sense that it is multiplied by the external field V_i^a which is not constant. On the other hand (11) is $SO(1,9)$ Lorentz invariant and this invariance can be used to eliminate this unwanted dependence on V_i^a. For a generic $SO(1,9)$ transformation $\Lambda^a{}_b$ we have $I(V, \Omega, y) = I(V^\Lambda, \Omega^\Lambda, y^\Lambda)$, and changing integration variable from y to y^Λ we can replace $I(V, \Omega, y)$ with $I(V^\Lambda, \Omega^\Lambda, y)$. But, since the measure $\int \{\mathcal{D}y\}$ is not invariant, this change of variable results in the appearance of $\Gamma_{WZ}(\Lambda)$ as given in (10). Therefore the integration over the fermionic y^α results in an effective action, Γ_F, which can be written as

$$\Gamma_F = \Gamma_0 - \Gamma_{WZ}(\Lambda) \tag{12}$$

where

$$e^{i\Gamma_0} = \int \{\mathcal{D}y\} e^{iI(V^\Lambda, \Omega^\Lambda, y)} \,. \tag{13}$$

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It remains to choose an appropriate $\Lambda^a{}_b$. For this purpose we introduce eight $SO(1,9)$ Lorentz vectors, N_a^r $(r = 2, ..., 9)$ satisfying

$$N_a^r N^{as} = -\delta^{rs}$$
$$N_a^r V_j^a = 0 \tag{14}$$

and choose

$$\Lambda^{\widehat{a}}{}_a = e^{-\phi} \widetilde{e}^{j\,\widehat{a}} V_{ja}$$
$$\Lambda^r{}_a = N_a^r \quad (r = 2, ..., 9) \tag{15}$$
$$\widetilde{e}_\pm^j = e^{\pm\phi} e_\pm^j \ .$$

Due to (3), (14) and $\widetilde{e}_a^i \widetilde{e}_b^j g_{ij} = \eta_{\widehat{a}\,\widehat{b}}$, this Λ is indeed an element of $SO(1,9)$, in that $\Lambda^a{}_b \Lambda^c{}_d \eta^{bd} = \eta^{ac}$. With this choice the kinetic term for y^α becomes indeed canonical

$$I(V^\Lambda, \Omega^\Lambda, y) = \frac{1}{4} \int d^2\sigma \sqrt{g} \ e_+^j \ y \ \Gamma_- \left(\partial_j - \frac{1}{4} \Gamma_{rs} W_j{}^{rs} \right) y \ , \tag{16}$$

where $\Gamma_- \equiv (\Gamma_0 - \Gamma_1)$ is a constant matrix and projects out just eight of the sixteen $y's$, and $W_j{}^{rs} \equiv N_a^s(\partial_j N^{ar} - \Omega_j{}^{ab} N_b^r)$ is Weyl, $SO(1,1)$ and $SO(1,9)$ *invariant* and does, therefore, not affect the corresponding anomalies. The relevant contribution to Γ_0 is therefore just given by $8 \ \ell n^{1/2} \det(\sqrt{g}\,\partial_+)$ which is equal to

$$\Gamma_0 = \frac{1}{48\pi} \int d^2\sigma \sqrt{g} \ tr \left(D_-\omega_+ \frac{1}{\Box} D_-\omega_+ \right) . \tag{17}$$

Here we defined $\omega_{\pm\widehat{a}\,\widehat{b}} = \omega_\pm \varepsilon_{\widehat{a}\,\widehat{b}}$. Under worldsheet Lorentz and Weyl transformations we have

$$\delta\Gamma_0 = -\frac{1}{24\pi} \int d^2\sigma \sqrt{g} \ tr \left(\partial_-(\ell - \lambda)\omega_+ \right) \tag{18}$$

$$-\delta\Gamma_{WZ} = -\frac{1}{8\pi} \int d^2\sigma \sqrt{g} \ tr \left(\partial_-(\ell - \lambda)\omega_+ \right) \ , \tag{19}$$

where we wrote $\lambda_{\widehat{a}\,\widehat{b}} = \varepsilon_{\widehat{a}\,\widehat{b}} \lambda, \ell_{\widehat{a}\,\widehat{b}} = \varepsilon_{\widehat{a}\,\widehat{b}} \ell$. The evaluation of $\delta\Gamma_{WZ}$ is long but straightforward. One has to use (6) together with (9), and the decomposition

$$\omega_{+\widehat{a}\,\widehat{b}} = e^{-2\phi} \left(V_{[\widehat{b}}^a \partial_+ V_{\widehat{a}]a} - \Omega_{+ab} V_{\widehat{b}}^a V_{\widehat{a}}^b \right) + \left(\frac{1}{2} e^{-2\phi} D_+ V_-^a V_{+a} - \partial_+\phi \right) \varepsilon_{\widehat{a}\,\widehat{b}}$$

which follows from the embedding equation (3).

We see that the effect of the Wess–Zumino term is just to quadruplicate the "naïf" result, $\delta\Gamma_0$, which corresponds to the Weyl–Lorentz anomaly of just

eight quantum $\vartheta's$. The contribution of the Wess–Zumino term, which is actually essential for the cancellation of the total anomaly, is missed in non–covariant perturbative approaches, refs. [1].

The relevant contribution to the effective action of the bosonic coordinates and the ghost fields is standard ($D = 10$)

$$\Gamma_{y^a,b,c} = \frac{D - 26}{96\pi} \int d^2\sigma\sqrt{g}\, \mathcal{R}^{(0)} \frac{1}{\Box}\, \mathcal{R}^{(0)} \ , \tag{20}$$

whose variation under Weyl and $SO(1,1)$ is just

$$\delta\Gamma_{y^a,b,c} = \frac{1}{3\pi} \int d^2\sigma\sqrt{g}\, tr\left((D_-\omega_+ - D_+\omega_-)\lambda\right) \ .$$

Summing up $\delta\Gamma_{y^a,b,c}$, $\delta\Gamma_F$ and the variation of the local term $\frac{1}{6\pi} \int d^2\sigma\sqrt{g}\, tr(\omega_+\omega_-)$ one gets for the total Weyl–Lorentz anomaly due to the fields (y^α, y^a, b, c)

$$\mathcal{A}_{\lambda,\ell} = \frac{1}{6\pi} \int d^2\sigma\sqrt{g}\, tr\left(\partial_+(\ell + \lambda)\omega_-\right) \ . \tag{21}$$

The corresponding κ–anomaly can be determined by enforcing the consistency condition on the complete anomaly, $\mathcal{A} = \mathcal{A}_{\lambda,\ell} + \mathcal{A}_\kappa$, $\Omega\mathcal{A} = 0$. This determines $\mathcal{A}$ as

$$\mathcal{A} = \frac{1}{6\pi} \int d^2\sigma\sqrt{g}\, tr\left(\partial_+(\ell + \lambda)\omega_- - 2\omega_-\omega_- V_+^\alpha\kappa_\alpha\right) \ . \tag{22}$$

So far we have set the heterotic fermions to zero. If they are present, eq. (3) gets modified to

$$V_i^a V_{ja} = g_{ij}e^{2\phi} + \frac{1}{4}e_{-i}e_{-j}\psi_0(\partial_+ - A_+)\psi_0 \ , \tag{23}$$

where ψ_0 are the classical heterotic fermions, and in this case Λ, as given in (15), does no longer belong to $SO(1,9)$. However, by defining a modified metric, g_{ij}^*, through

$$\begin{aligned} e_{+j}^* &= e_{+j} + \frac{1}{4}\, e^{-2\phi}e_{-j}\psi_0(\partial_+ - A_+)\psi_0 \\ e_{-j}^* &= e_{-j} \ , \end{aligned} \tag{24}$$

one can rewrite (23) as $V_i^a V_{ja} = g_{ij}^*e^{2\phi}$ and, using this, one can again construct a $\Lambda^* \in SO(1,9)$ along the same lines which brought to (15). The shift in (24) cannot modify the Weyl–Lorentz anomaly, (21), gotten by the functional integration over (y^α, y^a, b, c), but only the κ–partner, by a term quadratic in ψ_0. We can again

enforce the Wess–Zumino consistency condition on $\mathcal{A}_1 = \mathcal{A} + \mathcal{A}'_\kappa$, $\Omega\,\mathcal{A}_1 = 0$, which gives now:

$$\mathcal{A}_1 = \frac{1}{6\pi} \int d^2\sigma \sqrt{g}\; tr\left(\partial_+(\ell + \lambda)\omega_- - 2\omega_-\omega_-\left(V_+{}^\alpha - \frac{1}{2}\,\psi_0\chi^\alpha\psi_0\right)\kappa_\alpha\right)\; . \quad (25)$$

The contribution of the 32 quantum heterotic fermions to the effective action (for $A_- = 0$) is standard and corresponds to

$$\Gamma_\psi = 32\; \ell n^{1/2}\; \det\; (\sqrt{g}\,\partial_-) = \frac{1}{12\pi} \int d^2\sigma \sqrt{g}\; tr\left(D_+\omega_-\frac{1}{\square}D_+\omega_-\right)\; . \quad (26)$$

Since we have the following total variations:

$$\delta\omega_- = \partial_-(\ell + \lambda) + (\lambda - \ell)\omega_-$$

$$\delta\left(\sqrt{g}\,D_+\omega_-\right) = \sqrt{g}\;\left(\square\,(\ell + \lambda) - 4D_-\left(\omega_-\left(V_+{}^\alpha - \frac{1}{2}\,\psi_0\chi^\alpha\psi_0\right)\kappa_\alpha\right)\right)$$

$$\square = D_+\partial_- = D_-\partial_+\; ,$$

one can easily compute $\mathcal{A}_2 = \delta\Gamma_\psi$ and verify that indeed

$$\mathcal{A}_1 + \mathcal{A}_2 = 0\; .$$

4. Some final remarks

Our procedure reveals a connection between the $SO(1,1)$ and $SO(1,9)$ anomalies which emerges as follows. The invariant polynomial corresponding to the target Lorentz anomaly (5) is given by $X_4^L(\mathrm{R}) = \frac{1}{8\pi}tr(\mathrm{RR})$ where R is the $D = 10$ Lorentz curvature two–form. On the other hand, the "naif" contribution of the eight physical y^α to the $SO(1,1)$ anomaly, eq. (18), corresponds to the invariant polynomial $X_4^{(0)}(\mathcal{R}) = -\frac{1}{24\pi}tr(\mathcal{R}\mathcal{R})$, where $\mathcal{R}$ is the $d = 2$ Lorentz curvature two–form. What we have shown is that the total $d = 2$ anomaly polynomial, corresponding to the y^α, is given by $X_4^y(\mathcal{R}) = X_4^{(0)}(\mathcal{R}) - X_4^L(\mathcal{R}) = -\frac{1}{6\pi}\,tr(\mathcal{R}\mathcal{R})$. The contribution to X_4 from the ψ is just (see (26)) $X_4^\psi(\mathcal{R}) = \frac{1}{6\pi}\,tr(\mathcal{R}\mathcal{R})$, and $X_4^\psi + X_4^y = 0$.

Our procedure for computing anomalies required the introduction of eight $SO(1,9)$ vectors, N_a^r $(r = 2, ..., 9)$, which span the (eight–dimensional) space orthogonal to the V_j^a. Classically these base vectors are defined only up to an

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(extrinsic) local $SO(8)$ rotation, $\delta N_a^r = \ell^{rs} N_{as}$, where $\ell^{rs} = -\ell^{sr}$. For consistency the quantum effective action should depend only on the orthogonal space but not on the particular basis $\{N_a^r\}$ we have chosen. It can, actually, be verified that neither Γ_0 nor Γ_{WZ} are $SO(8)$ invariant, but that $\Gamma_F = \Gamma_0 - \Gamma_{WZ}$ is indeed invariant, as expected.

Let us also notice that under target–space $SO(1,9)$ rotations we have $\delta_L(\Gamma_0 - \Gamma_{WZ}) = -\delta_L \Gamma_{WZ} = \frac{1}{8\pi} \int d^2\sigma \sqrt{g} \; tr(\partial_- L \widetilde{\Omega}_+)$ which reproduces correctly the anomaly (5).

What we have considered in this paper are the "genuine string" worldsheet anomalies in the heterotic string sigma–model, i.e. those anomalies which survive (in non critical dimensions) even when the target fields are switched off. The "genuine sigma–model" anomalies at one loop, which go to zero when the target fields go to zero, have been determined in ref. [6]. For completeness we recall the result:

$$\mathcal{A}_\sigma = -\frac{1}{16\pi} \int d^2\sigma \varepsilon^{ij} V_i^A V_j^B \Delta^\gamma (\omega_{3YM} - \omega_{3L})_{\gamma BA} \,, \tag{27}$$

where ω_{3YM} and ω_{3L} are the Yang–Mills and $SO(1,9)$–Lorentz Chern–Simons three-superforms satisfying $d\omega_{3L} = tr(\mathrm{RR})$, $d\omega_{3YM} = tr(\mathrm{FF})$. $\mathcal{A}_\sigma$, which is a pure κ–anomaly, is cancelled by defining the generalized supercurvature

$$H = dB + \frac{\alpha'}{4}(\omega_{3YM} - \omega_{3L}) \tag{28}$$

and imposing on it, rather than on dB, the constraints (IV, V) in the appendix. The Bianchi identity associated to (28),

$$dH = \frac{\alpha'}{4}(tr(\mathrm{FF}) - tr(\mathrm{RR})),$$

can then be consistently solved in superspace at first order in α', see refs.[6,9]. Eq. (28) implies also an anomalous transformation law for B, which is just the right one to cancel the $SO(1,9)$ and $SO(32)$ anomalies associated to the anomaly polynomial

$$X_4 = \frac{1}{8\pi}(tr(\mathrm{RR}) - tr(\mathrm{FF})).$$

APPENDIX

1. The action (1) is κ–invariant if the target space fields satisfy suitable constraints. If we define the target space super–differential $d = dZ^M \frac{\partial}{\partial Z^M}$ and $E^A = dZ^M E_M{}^A$ and $T^A = dE^A + E^B \Omega_B{}^A \equiv \frac{1}{2} E^B E^C T_{CB}{}^A$, $F = dA + AA \equiv \frac{1}{2} E^A E^B F_{BA}$, $H = dB \equiv \frac{1}{6} E^A E^B E^C H_{CBA}$ these constraints are given by

$$T_{\alpha\beta}{}^a = 2(\Gamma^a)_{\alpha\beta} \quad (I)$$

$$T_{a\alpha}{}^b = 0 \qquad (II)$$

$$F_{\alpha\beta} = 0 \qquad (III)$$

$$H_{\alpha\beta\gamma} = 0 = H_{ab\alpha} \quad (IV)$$

$$H_{a\alpha\beta} = 2(\Gamma_a)_{\alpha\beta}. \quad (V)$$

(III) implies in particular that $F_{a\alpha} = 2(\Gamma_a)_{\alpha\beta}\chi^\beta$, where χ^β is the gluino, with values in the Lie algebra of $SO(32)$, and (I, II) imply that $T_{\alpha\beta}{}^\gamma = 2\delta^\gamma_{(\alpha}\lambda_{\beta)} - (\Gamma^a)_{\alpha\beta}(\Gamma_a)^{\gamma\delta}\lambda_\delta$, where $\lambda_\alpha = D_\alpha\varphi$, φ being the dilaton superfield.

2. The BRST transformations for the ghost fields which, together with (2), give rise to an on–shell nihilpotent BRST operator, $\Omega^2 = 0$, are given by:

$$\delta\ell = -c^j \partial_j \ell - (\partial_- + \omega_-)(\kappa V\!\!\!/_- \kappa)$$

$$\delta\lambda = -c^j \partial_j \lambda + (\partial_- - \omega_-)(\kappa V\!\!\!/_- \kappa)$$

$$\delta c^i = -c^j \partial_j c^i + 2e^i_- (\kappa V\!\!\!/_- \kappa)$$

$$\delta\kappa_\alpha = -c^j \partial_j \kappa_\alpha + (\lambda - \ell)\kappa_\alpha - (V^a_- \lambda_\alpha + (\Gamma^a)_{\alpha\gamma}V^\gamma_-)(\kappa\Gamma_a\kappa)$$

$$+ (4V^\gamma_- \kappa_\gamma - \kappa V\!\!\!/_- \lambda)\kappa_\alpha - \Delta^\varepsilon \Omega_{\varepsilon\alpha}{}^\gamma \kappa_\gamma.$$

3. The quantity $T_+{}^{ab}$ in (6) can be read off from eq. (56) of ref. [6] and is given by

$$T_+{}^{ab} = \widetilde{T}^{abc}V_{+c} + e^{-2\phi}\widetilde{T}^{[a}{}_{cd}V_+{}^c V_-{}^d V_+{}^{b]}$$

where the completely antisymmetric tensor $\widetilde{T}^{abc}$ is given by

$$\widetilde{T}^{abc} = T^{abc} + e^{-2\phi}\left(\frac{1}{2}(\Gamma^{abc})_{\alpha\beta}V_+{}^\alpha V_-{}^\beta - \frac{1}{16}(\Gamma_g \Gamma^{abc})_\gamma{}^\alpha V^g_- V_+{}^\beta T_{\alpha\beta}{}^\gamma\right),$$

and T^{abc} is the supertorsion.

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